Teacher Questioning to Support Young Students to Interpret and Explain Their Critical Mathematical Thinking

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This paper examines the types of teacher questions that assist young students to interpret and explain their critical mathematical thinking (CMT). Focusing on students who enter their first year of formal schooling (aged 5–6 years), this paper draws on data from a one-on-one task based clinical interview conducted with 16 students. Teacher questioning data were analysed for question type (probing, factual, guiding) and further analysed against the conceptualized critical mathematical thinking framework for young students. Findings indicated that when teachers used clarifying, noting relationships, and offering opinions style probing questions, young students were supported to interpret and explain their critical mathematical thinking.

The ability to apply mathematical thinking in a critical way is essential to our students' future success in education and employability (Heard et al., 2020; National Council of Teachers of Mathematics (NCTM), 2000). Research, policy, and curricula have indicated the importance of including critical thinking in the discipline of mathematics starting in the early years of schooling (NCTM, 2000; Wood et al., 2006). However, to date, little research has been undertaken to better understand how critical thinking and mathematical thinking amalgamate, which has left this area poorly defined and under researched. Consequently, this has impacted on classroom practices, and recent reports indicate that typically educators teach generalised critical thinking skills without a theoretical underpinning and without examples for domain specific areas (Sweller, 2022). This eventuated in the emergence of a new term, Critical Mathematical Thinking (CMT) that brings together critical thinking and mathematical thinking in a domain specific way (Monteleone, 2021). A recent systematic review of the literature, drawing on both mathematical thinking and critical thinking, led to the development of a theoretically informed conceptual framework, the Critical Mathematical Thinking Framework for Young Students (CMTFYS) (Monteleone, 2021) to better understand the key features of CMT for young learners.

The question remains as to how teachers can support young students to interpret and explain their critical mathematical thinking. It is well established in the literature, that the role of teacher questioning is central to supporting young learners to elicit their thinking (Franke et al., 2009; Sahin & Kulm, 2008), however, it is not clear what examples of teacher questions posed best support young students to exhibit domain specific thinking such as CMT. Thus, the focus of this paper is to begin to identify the particular types of teacher questions that help young students to interpret and explain their CMT as they participate in mathematics learning experiences.

Critical Mathematical Thinking For Young Students' Conceptual Framework

While the literature surrounding critical thinking (e.g., Facione, 2011) and mathematical thinking (e.g., Clements & Sarama, 2014) are diverse, there are also many commonalities. In the development of the conceptual framework, seminal work were reviewed and five critical mathematical thinking themes with supporting sub-themes emerged from the literature (Monteleone, 2021). The themes, *Interpreting* (e.g., Facione & Facione, 2008), *Analysing* (e.g., Lai, 2011), and *Creating* (e.g., Facione, 2011) emerged from the critical thinking literature while *Evaluating* (e.g., Wood et al., 2006) and *Explaining* (e.g., Diezmann et al., 2001)

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emerged from both critical thinking and mathematical thinking literature. Findings of a larger study identified that the CMTFYS provides the definition of CMT capabilities in young students (Monteleone, 2021), and offers a unique contribution to Critical Mathematical Thinking as a term. Indicating that CMT is more diverse from mathematical thinking alone. Figure 1 presents the CMTFYS which includes the five themes and 14 sub-themes. In this figure the themes are in bold and the sub-themes are italicized. The bolded themes and italicized sub-themes offer a unique contribution of critical thinking literature to this framework.

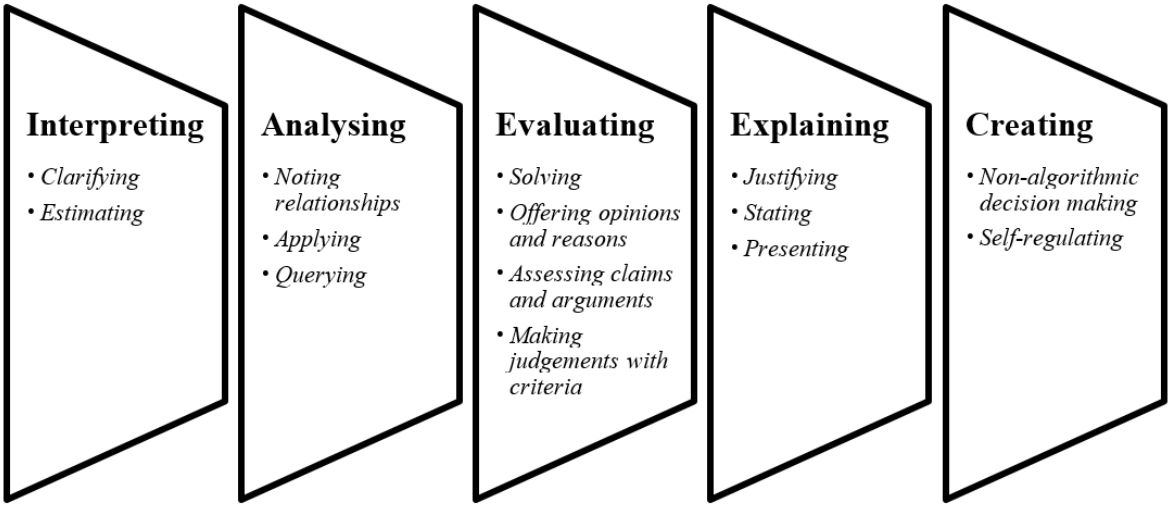

Figure 1. Critical mathematical thinking for young students' conceptual framework (CMTFYS) (Monteleone, 2021).

Further investigation is needed to understand the role of the teacher to support students to elicit their CMT. Therefore, for the purpose of this paper, the CMTFYS conceptual framework serves as a lens to help classify teacher questions and provided a platform to exemplify the types of questions that can be used to support young learners.

The Role of the Teacher Questioning in Mathematics

It is known that within mathematics learning in the early years, the role of the teacher is central, especially in supporting young students to elicit their thinking (Franke et al., 2009). Literature informed approaches to teaching mathematics that appear to be the most beneficial to young learners require teachers to: (i) engage young students by integrating mathematical concepts; (ii) embed problem solving; and (iii) allow for construction of ideas (Lessani et al., 2017). Adopting the mentioned teacher approaches has been found to provide a platform for students to engage in reasoning, creativity and allow for communication of their mathematical ideas (Wood et al., 2006). In order to enact this in classrooms with early years learners, teachers must engage young students in mathematical talk through the use of questioning. Encouraging student mathematical talk has been found to assist learners to engage in thinking processes, including justifying and reasoning about their approaches and solutions in class (Hunter & Anthony, 2011).

Teacher questioning plays an important role in promoting students' thinking in mathematics classrooms (Franke et al., 2009; Mata-Pereira & da Ponte, 2017; Sahin & Kulm, 2008). A range of teacher questioning used in the classroom supports student engagement in building mathematical understandings (Martino & Maher, 1999). Yet, research undertaken with grade two and three students identified that over 76% of teacher questions in mathematics

lessons were at best surface level, requiring a yes or no answer (Di Teodoro et al., 2011). It is evident that there continues to be an ongoing challenge with the types of and diversification of teacher questions used in mathematics classrooms.

In mathematics education, teacher questioning can be categorised in a myriad of ways; for example, *factual, probing* and *guiding* questions (Sahin & Kulm, 2008). *Factual questions* tend to provide very little information about a student's understanding of a concept or content and, are often lower order with little opportunity to discuss strategies with others (Sahin & Kulm, 2008). *Probing questions* have been found to extend student's understanding, knowledge, and mathematical thinking, moving students from low level to higher order thinking (Sahin & Kulm, 2008). Franke et al. (2009) recognises the need for probing questions as a way for teachers to gain further clarity about a student's explanation. *Guiding questions* are considered questions that direct students to derive concepts or procedures to solve problems (Mata-Pereira & da Ponte, 2017). Both probing and guiding questions have been found to best support students to display higher levels of mathematical thinking. It is important to note that a critique of the teaching question literature is that at times the focus is too narrow, only focusing on teacher questions and not fully understanding the impacts of this across an entire teacher-student conversation. Hence, it is important to understand the impact of the teacher question on the student learning.

Thus, to ascertain the role of teacher questioning in supporting young learners to elicit their CMT, the study was underpinned by the following research question:

What types of teacher questions help young students interpret and explain their CMT?

Research Design

The findings presented in this paper are drawn from a larger study that employed an explanatory mixed methods design (Creswell, 2013) to examine how young students elicit their critical mathematical thinking. The focus of this paper is on the qualitative data which was collected to better understand the statistical analysis in more depth, particularly with regards to the CMT capabilities of young students and the role of teacher questioning.

Participants and Context of the Study

In total, 161 Kindergarten students participated in the larger study (aged: 5 years 1 month to 6 years 8 months). These Kindergarten students were from three urban primary schools in NSW Australia and were in their first six months of formal schooling. The participating schools had similar demographical features, with the Index of Community Socio-educational Advantage levels between 1092–1112 (a score above 1000 indicates a high-level of socio-economic status). These schools had similar above average Australian results in National Numeracy assessments (NAPLAN).

There were different data collection stages within the larger study to narrow student participants. The reason for narrowing the student group was to identify young students who exhibited some CMT capabilities in mathematics lessons which was observed by the researcher and evidenced across quantitative measures. This smaller group would provide an opportunity to explore more deeply the types of CMT students display and the role of teacher questioning to support students to explain and interpret their CMT. In total, 16 students were identified to participate in interviews. This included nine male and seven female students. The 16 students were represented from each of the schools of the larger study (25% School A; 32% School B; 43% School C).

Data Collection Methods

To investigate the types of teacher questions that assist young students' to interpret and explain their CMT, the data collection methods included a task based one-on-one clinical interview. The interview consisted of eight learning experiences. The clinical interview process followed *Piaget's methode Clinique* (Hunting & Doig, 1997) that identifies the cognitive capabilities of a child within the learning social context. Previous studies in mathematics education have adopted this method to assess young children's mathematical learning (Clements & Sarama, 2014; Hunting & Doig, 1997; Warren et al., 2012). Of particular importance, is the balance of the researcher encouraging students to elaborate on their responses while refraining from steering students towards a desired answer (Miller, 2014; Warren et al., 2012). The dialogue between the student and the researcher supported the researcher to clarify, ask questions and pose problems during the interview.

The interview included learning experiences which were designed to: (i) begin with an open-ended question (Nicol & Bragg, 2009); (ii) provide multiple entry points for students (Jorgensen et al., 2010); (iii) use physical manipulatives (MacDonald & Lowrie, 2011); and (iv) cover a range of mathematical content appropriate for the age group. Table 1 presents examples of learning experiences from the clinical interview where the students more commonly drew on CMT pertaining to explaining and interpreting as delineated in Figure 1.

Table 1
Example Learning Experiences from the Clinical Interview

LE	Description of the learning experience
LE1	*Framed photo finding the middle* This is a framed photograph of Joey (*hold up frame*). I would like to hang this frame in the middle of a wall. Now, imagine this piece of paper is a blank wall *(hold up A3 paper)* and this is the picture frame I need to hang (*hold up smaller frame*). How can I hang this frame in the middle of the wall?
LE2	*Counting unseen items* This is a mini bean bag *(show mini bean bag).* It is filled with little beans like these (*show zip lock bag with some beans*). It's too tricky to count them one by one. Can you think of another way to find out how many beans are in this mini bean bag?
LE6	*Cubby house – identifying number of tiles required* I have just finished building a cubby house for my children at home *(show picture of the cubby house)*. I would like to put these tiles down on the floor of the cubby house *(show square tile)*. How can I work out how many tiles I need?

The interviews were administered in a context that was familiar to the student (e.g., in a breakout room). Each student interview took between 25–35 minutes to administer. All interviews were video recorded to capture both the student responses and the researcher questioning. These videos were downloaded and were later transcribed for data analysis.

Data Analysis

Analysis of the transcripts occurred in a three-step process. First, all student responses were coded using the CMTFYS to determine the types of CMT young students exhibited. Second, all teacher questions were coded as either: factual (Sahin & Kulm, 2008), probing (Franke et

al., 2009; Sahin & Kulm, 2008), or guiding questions (Mata-Pereira & da Ponte, 2017). Third, the coded teacher questions were re-analysed and deductively coded using the 14 sub-themes of the CMTFYS. It was important to code the student responses first to ensure these young learners were exhibiting CMT, before determining the teacher question (TQ) that supported this. Table 2 provides an example of the final coding of the researcher (R) and student (S23) conversation for Learning Experience One.

Table 2
Example of Coding for the Transcript of Learning Experience One

	Transcription	Alignment with the CMTFYS
R	Is there another way you can check?	*TQ: Probing – Clarifying*
S23	That there on the sides. There, that's the long way of them. And that is the length.	*Student CMT: Explaining- Justifying*
R	The length, I heard that very special word length. How can we check it*?*	*TQ: Probing – Clarifying*
S23	... If you put it with a measuring tape on the even number, you know it's the middle because the odd number that doesn't add up, it would totally be in the middle. It's the same length as the other, so that's the middle.	*Student CMT: Explaining - Justifying*
R	You could use the same strategy?	*TQ: Probing- Noting Relationships*
S23	… you could draw a line like that, two lines, above one line. Then make another one to show where your pictures standing, and then put the other line here ... So, you can do those lines. And then you put the photo on.	*Student CMT: Explaining- Stating*
R	Can you show me*?*	*TQ: Guiding - Assessing*
S23	And then what you do is you hang it up like that. Then you'll know which is the middle.	*Student CMT: Explaining - Stating*

Results and Discussion

In total, 333 probing, guiding and factual questions were posed to the 16 students for each learning experience and across all interviews. Table 3 displays the percentage of the types of researcher questions used in the interview across the eight learning experiences.

Table 3
Percentage of Types of Researcher Questions used in Interview

Questions Type	Example question	Percentage
Probing	What do you think? Can you explain that to me?	59% (196/333)
Guiding	Can you show me? Is that the best way?	25% (84/333)
Factual	Which one do you think has more? Why are both sides halves?	16% (53/333)

The data evidenced that all types of teacher questions (factual, probing, guiding) were used across the interviews. This aligns with past research (Franke et al., 2009; Mata-Pereira & da

Ponte, 2017; Sahin & Kulm, 2008). In addition to this, the analysis revealed that probing questions were the most used to support young students to elicit their CMT. While guiding and factual questions were also used by the researcher in the interview, it appeared that these types of questions were not as common. Thus, the following section provides further understanding of probing questions, specifically, how this type of questioning aligned with the CMTFYS and how these questions support young students to interpret and explain their CMT.

Using Probing Teacher Questioning to Support Students to Interpret and Explain their CMT

Further analysis of the probing questions utilised, aligned to the CMTFYS, revealed that the probing questions aligned with three types of CMT. These included: clarifying, which emerges from the critical thinking literature (CT); and noting relationships and offering opinions, which align with both critical thinking (CT); and mathematical thinking (MT) literature. Table 4 provides the: (i) frequency percentage and type of the probing questions used by the researcher; (ii) a description of the type of probing question used during the conversation; (iii) an example of the researcher questions drawn from the transcripts; and (iv) an example of the student response showing student's interpreting and explaining their CMT.

Table 4
Probing Researcher Questions to Support Young Students to elicit their CMT.

Type of question	Description of Question Type	Example of Researcher Question	Example of student CMT
Clarifying (53%) [CT]	The researcher rephrased or re-used the students' terms to gain further insight into their CMT.	*How do you know they're the same?*	"You need to do this [S using arm spans to gesture equality] so you can see it is the same." *(Student CMT Interpreting – Clarifying)*
Noting Relationships (37%) [CT and MT]	The researcher asked questions to gain further understanding of the relationship's young students were seeing across mathematical concepts.	*How do you know it is the middle?*	"How about I measure it, we bring the pencil here [moves pencil to measure to show the location of the middle]." *(Student CMT: Interpreting – Estimating)*
Offering Opinions (10%) [CT and MT]	To redirect student thinking, the researcher included questions to support young students to provide opinions about their thinking.	*Can you tell me what that would look like?*	"I am pretending this is a measuring tape …. Then you get five tiles … then you lay them out in a row of five." *(Student CMT Explaining - presenting)*

The analysis of data revealed that probing clarifying teacher questions were the most commonly used in the interviews, followed by probing noting relationships and then probing offering opinions. It is noted in previous studies that probing questions can support students to move from lower level to higher levels of thinking (Franke et al., 2009; Sahin & Kulm, 2008),

however, the use of the CMTFYS provides new insight into understanding the role of teacher questions, in particular, how specific questions during conversations can help students further interpret and explain their CMT. The descriptions offered for each type of teacher probing question, in Table 4, extends on the research by Franke et al. (2009) and Sahin and Kulm, (2008) by providing specific teacher questions to be used during a conversation. For example, teachers can pose *probing clarifying questions* to young students.

Research indicates that supporting young students to communicate their ideas (Wood et al., 2006) and engage in mathematical talk to articulate their thinking (Hunter & Anthony, 2011) is deemed important. More so, in order to prepare students to engage in CMT, support for teachers is required to assist in moving beyond generic approaches adopted in their mathematics classroom (Sweller, 2022). If young learners need to display CMT, then teachers need to pose specific questions that align with themes and sub-themes from both the critical thinking and mathematical thinking literature. The findings of this study begin to address this gap by providing a domain specific theoretical framework (CMTFYS) and begins to align teacher practices, in particular the questions they ask to this framework. By beginning to understand the types of teacher questions that support students to interpret and explain their CMT, both the talk used by the teacher and students in mathematics interviews has the potential to provide opportunities in broader classroom discussions (Monteleone, 2021).

Conclusion

The CMTFYS is a platform for teachers to understand how young students can engage with CMT in early years classrooms. The findings of this study suggest there are particular types of questions that can be presented to young students to assist them to interpret and explain their CMT while engaging with mathematical learning experiences. It appeared that teachers could ask probing questions that aligned more closely to support students in *clarifying, noting relationships* and *offering opinions*. This adds to the field by demonstrating that it appears there is a nuanced difference in how probing questions can be asked for young students to elicit their CMT. Thus, the CMTFYS is a new framework that can assist teachers to identify student's critical mathematical thinking and ascertain the types of questions teachers can use to help young students exhibit CMT.

Critical Mathematical Thinking is a new term in mathematics education and an emerging field of research. The findings from this study, informed by the CMTFYS conceptual framework, provide a contribution to the literature in two ways. First, how CMT is conceptualised in early years mathematics and second, the types of teacher questions that can support young learners to interpret and explain their CMT. It is evident that more work is to occur to apply this framework across the school setting (e.g., middle and upper primary classrooms; secondary classrooms) and with larger samples of students. The findings have potential to support teachers to reflect on their own teacher questioning techniques by considering how their questions align with the CMTFYS. In addition to this, teachers can consider how their questioning can shape the conversations with young students to elicit CMT. It is apparent that ongoing support is needed to assist teachers to understand CMT in a domain specific way, which should include the provision of evidence informed teacher practices to support CMT in their classrooms.

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